Bernoulli’s Theorem and the Hazen-Williams Equation: *rapid determination of diameter using the Hazen-Williams Equation*

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Bernoulli’s Equation is the theoretical basis for describing the physical phenomena of fluid flows.

In a pipeline, the energy of a water particle is a strict combination of potential energy, kinetic energy and pressure energy. In nature, “nothing is lost and nothing is created.” Therefore, the energy of a water particle passing through point 1 will be the same as when it passes through point 2 or any other point in the pipeline.

As an equation, this theorem may be expressed as follows:

Potential energy + pressure energy + kinetic energy = constant (1)

Or, in a more elaborate form:

$$h + \frac{P}{\rho} + \frac{V^2}{2g} = \text{Constante} \qquad (2)$$

where: h = height of the particle (in metres)
P = pressure (in pascals or newtons per m^2)
ρ = volume weight of the fluid (newtons per m^3)
V = velocity of the particle (in metres per second: m/s)
g = acceleration of gravity, namely 9.81 ms^{-2}

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Equation (2) may also be expressed in the following way for any two points (P1 and P2) in a pipeline.

$$h_1 + \frac{P_1}{\rho} + \frac{V_1^2}{2g} = h_2 + \frac{P_2}{\rho} + \frac{V_2^2}{2g} \quad (3)$$

In reality, energy losses through friction occur all along the trajectory. Such energy losses, or **friction losses**, enable the following equation to be written (4):

$$h_1 + \frac{P_1}{\rho} + \frac{V_1^2}{2g} = h_2 + \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Pch \quad (4)$$

Where **Pch** represents the loss of energy due to the friction of water particles both on the pipe walls and between themselves. This equation can be validated by 3 typical examples, illustrated by figures 1 and 2.

<u>First example (figure 1):</u>

The pipeline segment is turned completely off at the entrance to the water discharge point (fountain). Equation (4) may then be expressed as follows:

$$(H_{R_1} - H_{R_2}) = \frac{P_{R_2}}{\rho} \quad (5)$$

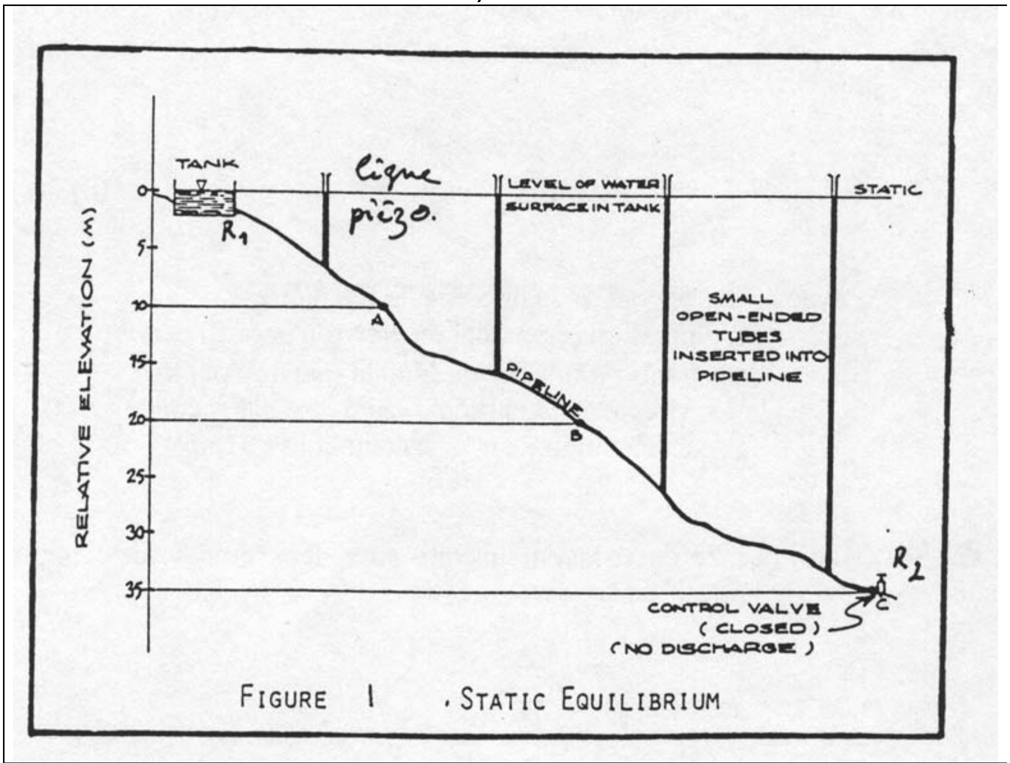

Figure 1: Illustration of Bernoulli formula. Pipeline turned off

(illustration extracted from: ***A Handbook of Gravity-flow Water Systems***, ref. p.7)

This means that all of the water energy at the low point of the water pipeline is in the form of pressure: that is:

$V_{R_1} = 0$ The velocity of the water in the water capture or tank can be considered as nil.

$P_{R_1} = 0$ At the capture point, the pressure at the entrance to the pipe is equal to atmospheric pressure.

$V_{R_2} = 0$ Since the valve is turned off.

The friction losses are nil: there is no flow and thus no friction.

Second example:

In this case the system is turned fully on, and the water flows freely from R_1 to R_2. Equation (4) becomes:

$$(h_{R_1} - h_{R_2}) = \frac{V_{R_2}^2}{2g} + Pch \qquad (6)$$

Since:

$V_{R_1} = 0$ The velocity of the water in the water capture or tank can be considered as nil (cf diagram 2).

$P_{R_1} = 0$ and $P_{R_2} = 0$ The pressure in R1 is nil (= to atmospheric pressure), the same as in R2 where the water flows freely. Since the valve is fully open, all of the water energy in R2 is transformed into velocity and friction (Pch).

Third example: The valve R2 is opened progressively. In this case, the equation becomes:

$$(h_{R_1} - h_{R_2}) = \frac{V_{R_2}^2}{2g} + \frac{P_{R_2}}{\rho} + Pch \qquad (7)$$

In this case, at point R2 the water flows at a speed less than in the previous case, since the valve is not completely open. The valve also maintains a certain pressure inside the pipe.

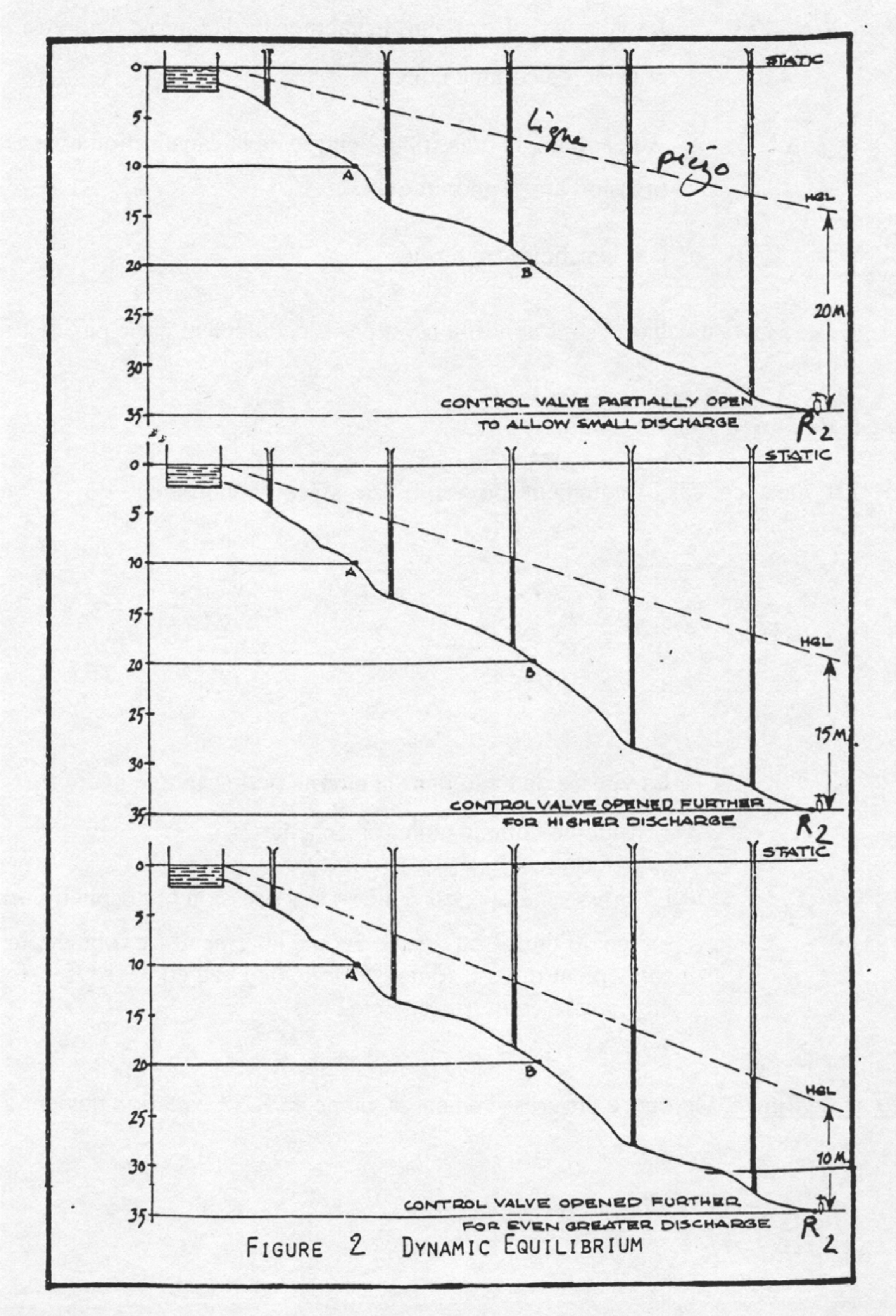

Figure 2: Bernoulli formula. Progressive opening of the pipeline
(illustration extracted from: ***A Handbook of Gravity-flow Water Systems,*** *ref. p.7)*

The Hazen-Williams equation: rapid determination of diameter using the Hazen-Williams Equation

Many researchers have worked on the experimental study of the determination of friction loss in a pipeline as a function of the state of the surface (and hence of friction) against the pipe walls. The formulae devised by Darcy, Colebrook and Hazen Williams may be mentioned as the most used.

The Hazen-Williams equation enables the determination of the maximum transportable flow by a length of pipeline according to the diameter chosen and the type of material used.

The example of calculations made for the Doge Larosso pipeline in Ethiopia shows the relevance of this hydraulic formula for optimising the design of the works to be constructed:

The Hazen-Williams formula is as follows:

$$Q_{max} = 0,2785\left(\frac{\Delta H}{L}\right)^{0,54} \cdot C_1 \cdot D_{in}^{2,63}$$

where: Q max is the maximum flow carried by the segment of pipeline (in m³ per second)

L	is the length of the segment (in m)
DH	is the difference in altitude between the two extremities of the segment (in m)
C1	is the friction coefficient of the material used

The following C1 values can be used:

Material	C1
PVC	145
New galvanised	130
Old galvanised	100

D*in* is the interior diameter (in m)
D*out* is the exterior diameter

In the Doge Larosso case, a segment between two distribution tanks needs to supply two discharge fountains (0.2 l/s per fountain); the maximum flow to be transported is 0.4l/s.
The difference in altitude between the two tanks is 40m, the length of the segment is 884m.

Scenario 1: the pipeline flow capacity is evaluated using a 2" pipe: D*out* = 50 mm and D*in* = 41 mm

Using Hazen-Williams:

$$Q_{max} = 0{,}2785.\left(\frac{40}{884}\right)^{0{,}54}.145.\ 0{,}041^{2{,}63}$$

$$Q_{max} = 0{,}00171.m^3/s = 1{,}71.l/s$$

It is clear that the 0.4l/s will be carried by the 2 inch pipeline, but it is no doubt possible to reduce the diameter without any adverse effect on the service provided.

Scenario 2: a 1.5" pipe is chosen (that is, having an exterior diameter of 32mm and interior diameter of 26.8mm).
The calculations using the Hazen-Williams formula show that the maximum flow carried in this case will be 0.56l/s.
The capacity of the segment is superior to the flow to be carried, so the choice is satisfactory.

In all cases the water velocity in the pipe can be calculated using the basic formula:

$$V = \frac{Q}{S}$$

where: V is the velocity of the water in the pipe (in m/s)
Q is the maximum flow of the pipeline (in m^3/s)
S is the segment of the pipeline (in m^2)

For scenario 1, the water velocity in the segment calculated in 2 inches diameter will produce a velocity of 0.3m/s.
This is less than the minimum recommended value of 0.7m/s. There is a high risk of sedimentation in the segment.

In the case of a choice of 32mm diameter, the water velocity will be 0.71m/s, which is sufficient to avoid sedimentation.

In the case of the Doge Larosso pipeline, the calculations show that the choice of 32mm piping is adequate. Cost comparisons in the two cases are interesting:

Cost 50 mm = 610 pipes x 74 EB + 10% plumbing = 49,654 EB (1 Ethiopian Birr ≅0.90FF)
Cost 32 mm = 610 pipes x 34 EB + 10% plumbing = 22,814 EB

The use of a formula enabling choice of the best design of a water pipeline is thus essential, for three main reasons:

- operation can be guaranteed without risk of sedimentation;
- significant savings can be achieved on materials (there was a cost reduction of over 50% in the case studied);
- pipeline maintenance will be more economical (parts and pipes will be cheaper). This is an important factor since these costs will fall to the beneficiaries of the aid, who generally have few resources.

Bibliography:

A Handbook of Gravity-flow Water Systems, Thomas D Jordan Jnr., Intermediate Technology Publications (I.T.) 1980 (250p.) - **highly recommended by Pierre Pioge**.
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